

Advances in the Use of MCDA Methods in Decision-Making

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Introduction : MCDM/MAUT/MCDA

- **Multiple Criteria Decision Making (MCDM) - Multiple Criteria Optimization**

Steuer, R. (1986) : *Multiple Criteria Optimization : Theory, Computation, and Applications*, John Wiley & Sons, New York.

- **MultiAttribute Utility Theory (MAUT)**

Keeney, R. and Raiffa, H. (1993) : *Decisions with Multiple Objectives : Preference and Value Tradeoffs*, Cambridge University Press, Cambridge.

- **Multiple Criteria Decision Aiding (MCDA)**

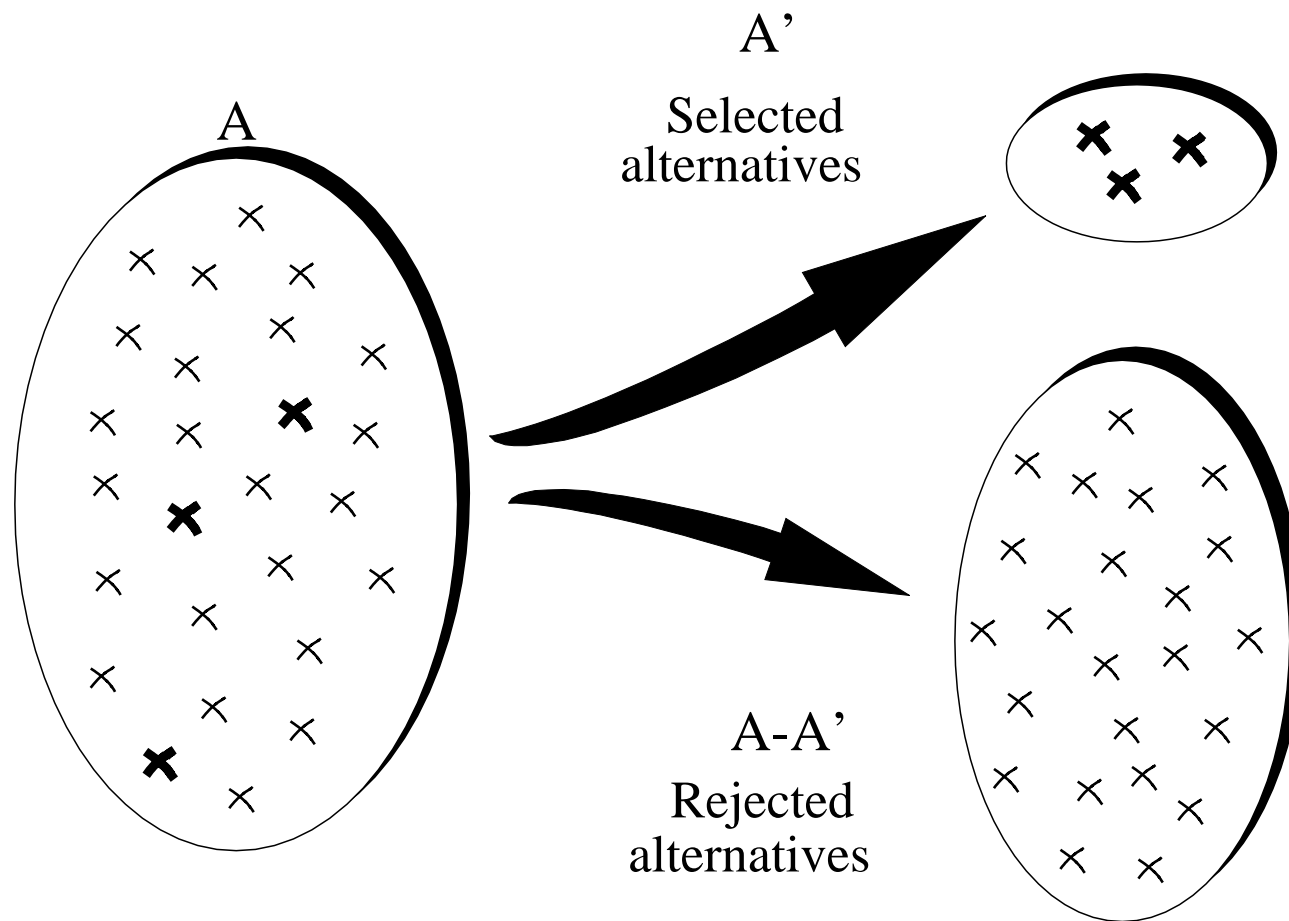
Roy, B. (1996) : *Multicriteria Methodology for Decision Aiding*, Kluwer Academic Publishers, Dordrecht.

Components of an MCDA model

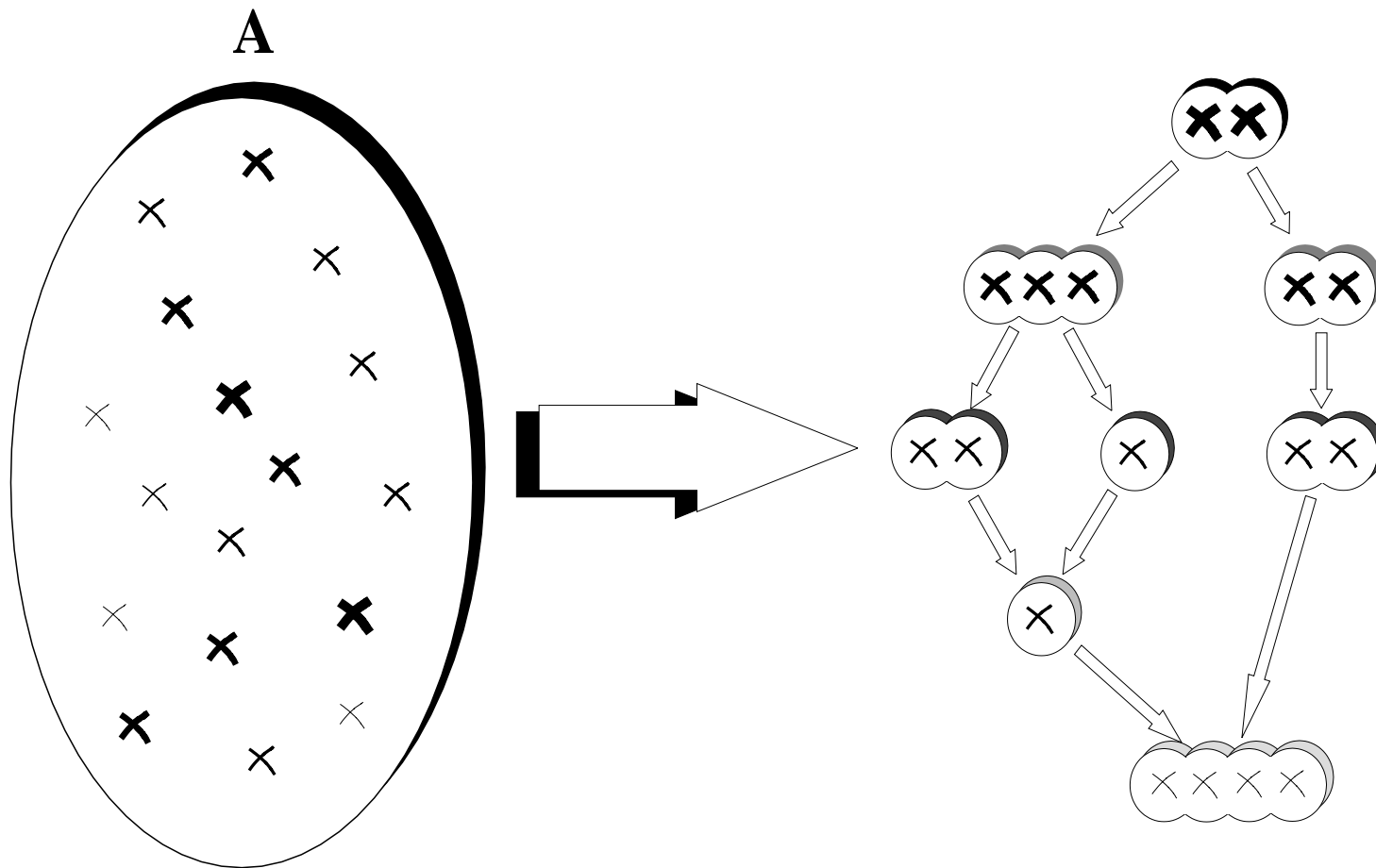
$$M_p = \langle A, D, G, \Omega, \mathcal{R} \rangle$$

- A is the set of **alternatives**.
- D is the set of **consequences** (attributes, effects, characteristics, ...).
- G is the **criteria** model.
- Ω is the **imperfect knowledge**.
- \mathcal{R} is the **aggregation procedure**.

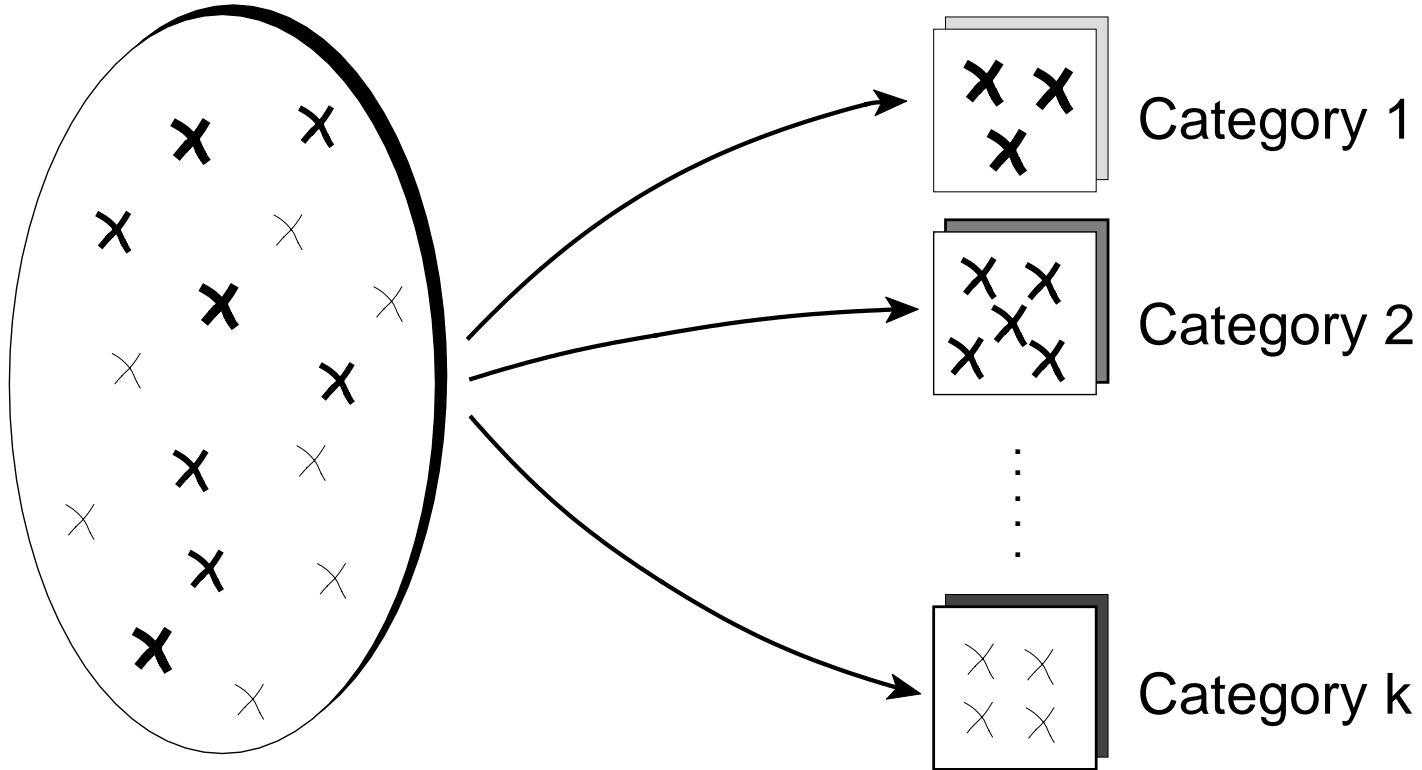
Problem statement : Choosing



Problem statement : Ranking



Problem statement : Sorting



Elementary Notation

- A finite set of **alternatives** $A = \{a_1, \dots, a_i, \dots, a_m\}$
- A finite set of **criteria** $G = \{g_1, \dots, g_j, \dots, g_n\}$
- A finite set of predefined and ordered **categories** $C = \{C_1, \dots, C_h, \dots, C_k\}$
- A finite set of **bounds** or profiles $\{b_1, \dots, b_h, \dots, b_{k-1}\}$
- $g_j(a_i)$ is the **evaluation** of the alternative a_i w.r.t. criterion g_j
- $u_j(g_j(a_i))$ is the **utility** of $g_j(a_i)$
- Let I denote the set of alternatives indices, F denote the set of criteria indices and K denote the set of categories indices
- The **“weights”** of criteria, w_1, \dots, w_n .

Descriptors and criteria : Definition and What for ?

Definition : A **descriptor** of impacts is an ordered set of plausible impact levels associated with a **key concern or criterion** (C. Bana e Costa and E. Beinart, 2005).

What for ?

1. **Operationalize** the appraisal of **impacts** (performances or consequences) of options in a key-concern, that is to measure (quantitatively or qualitatively) the degree to which the key- concern is satisfied.
2. **Describe**, as much as possible **objectively**, the **impacts** of alternatives with respect to that key-concern.
3. Better frame the evaluation model ; by **restricting** (whenever appropriate) the **range of impact** levels to a plausibility domain.
4. **Verify** the ordinal **independence** of the corresponding key-concern.

Descriptors and criteria : Types of descriptors

Depending on the context, alternative attributes may be more or less directly related with the key-concerns.

1. The levels of a **direct (natural)** descriptor directly reflect the effects ; such as the number of people affected by respiratory diseases.
2. The levels of an **indirect (proxy)** descriptor indicate causes more than effects ; such as the degree of concentration o air pollutants that cause respiratory diseases.
3. Remark 1 : **A natural descriptor is not necessarily more adequate than a proxy one.**
4. Remark 2 : Often **neither a direct nor an indirect** attribute exists which is appropriate to be used as the descriptor of the key-concern. We have to **construct** a descriptor (verbal descriptors of expected consequences, reference impact profiles, visual representations, indices, ...)

The OUTRANKING BASED METHODS : ELECTRE (PSEUDO-CRIT)

A **pseudo-criterion** is a function g_j associated with two threshold functions $q(g_j(\cdot))$ and $p(g_j(\cdot))$ satisfying the following conditions, for all $a, b \in A$:

$$q(g_j(b)) - q(g_j(a)) \geq g_j(a) - g_j(b) \quad (1)$$

$$p(g_j(b)) - p(g_j(a)) \geq g_j(a) - g_j(b), \quad (2)$$

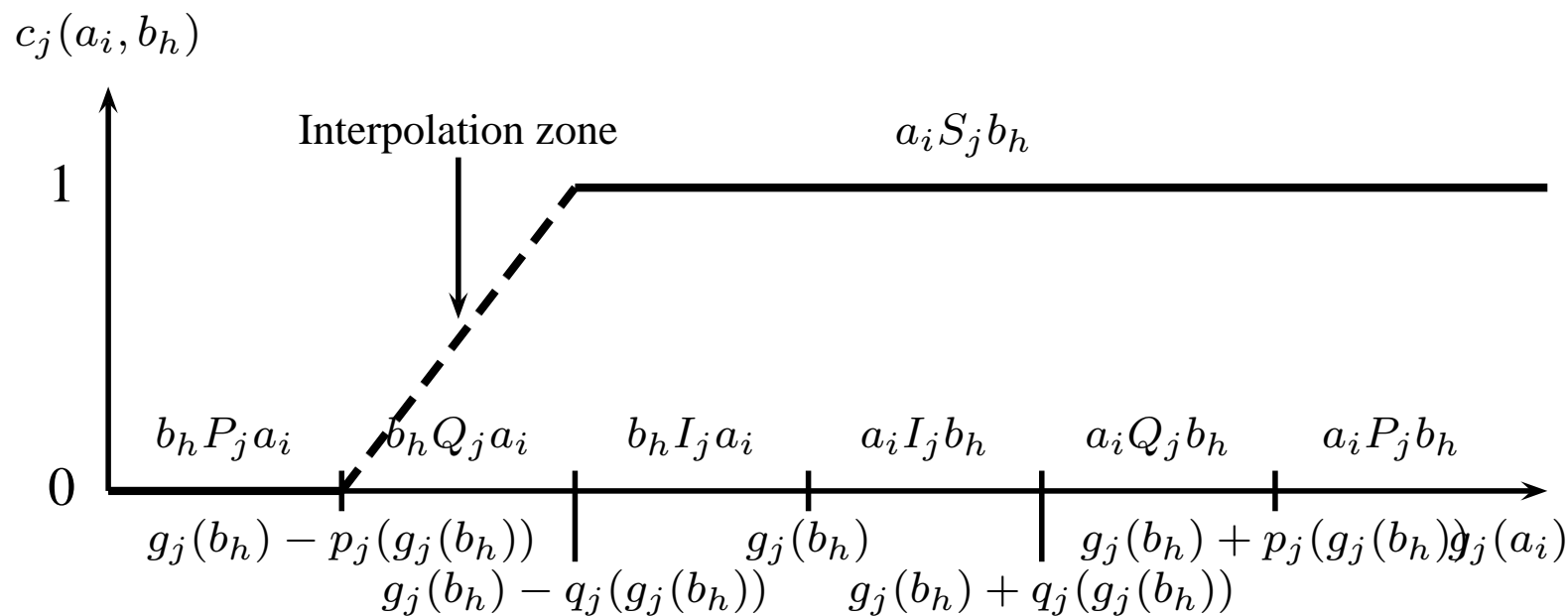
and such that, for all $a, b \in A$ with $g_j(a) \geq g_j(b)$:

$$aI_jb \Leftrightarrow g_j(a) \leq g_j(b) + q(g_j(b)) \quad (3)$$

$$aQ_jb \Leftrightarrow g_j(b) + q(g_j(b)) < g_j(a) \leq p(g_j(b)) \quad (4)$$

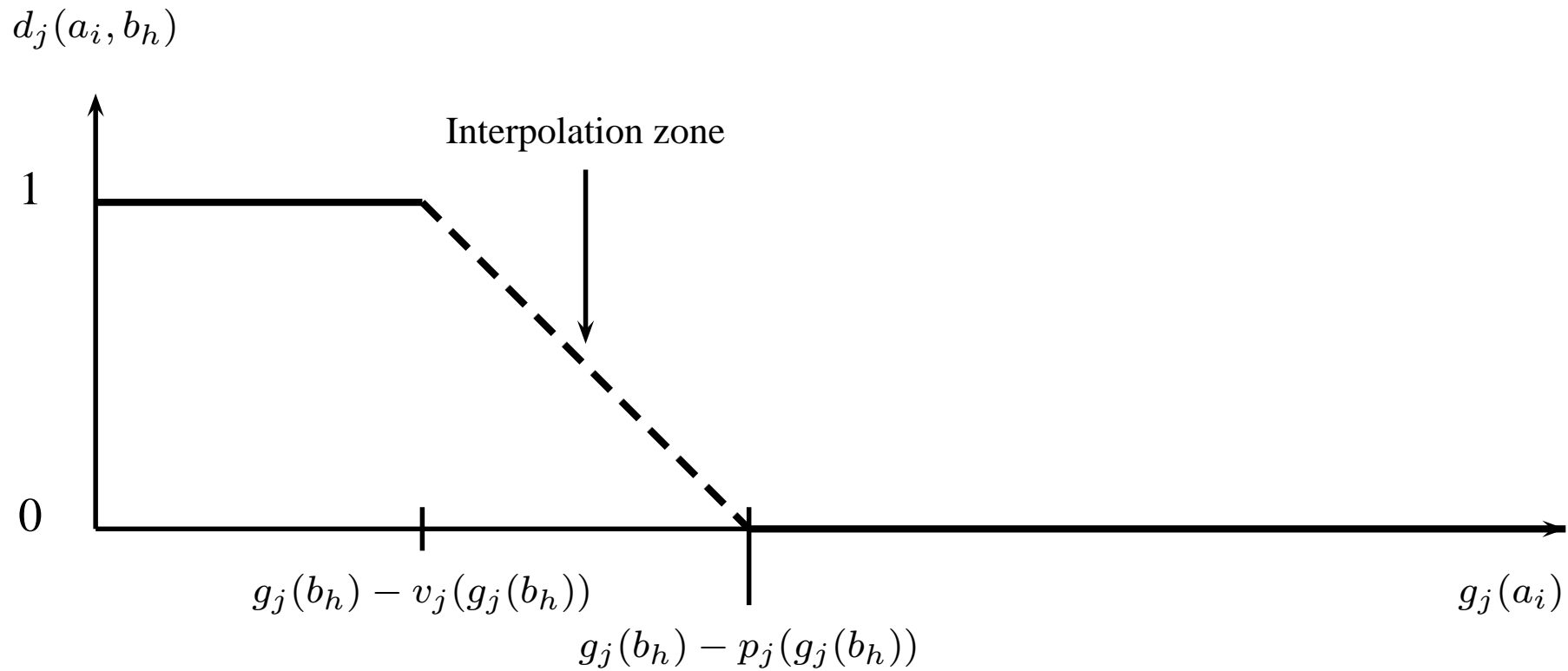
$$aP_jb \Leftrightarrow g_j(b) + p(g_j(b)) < g_j(a). \quad (5)$$

The OUTRANKING BASED METHODS : ELECTRE (CONCORD)

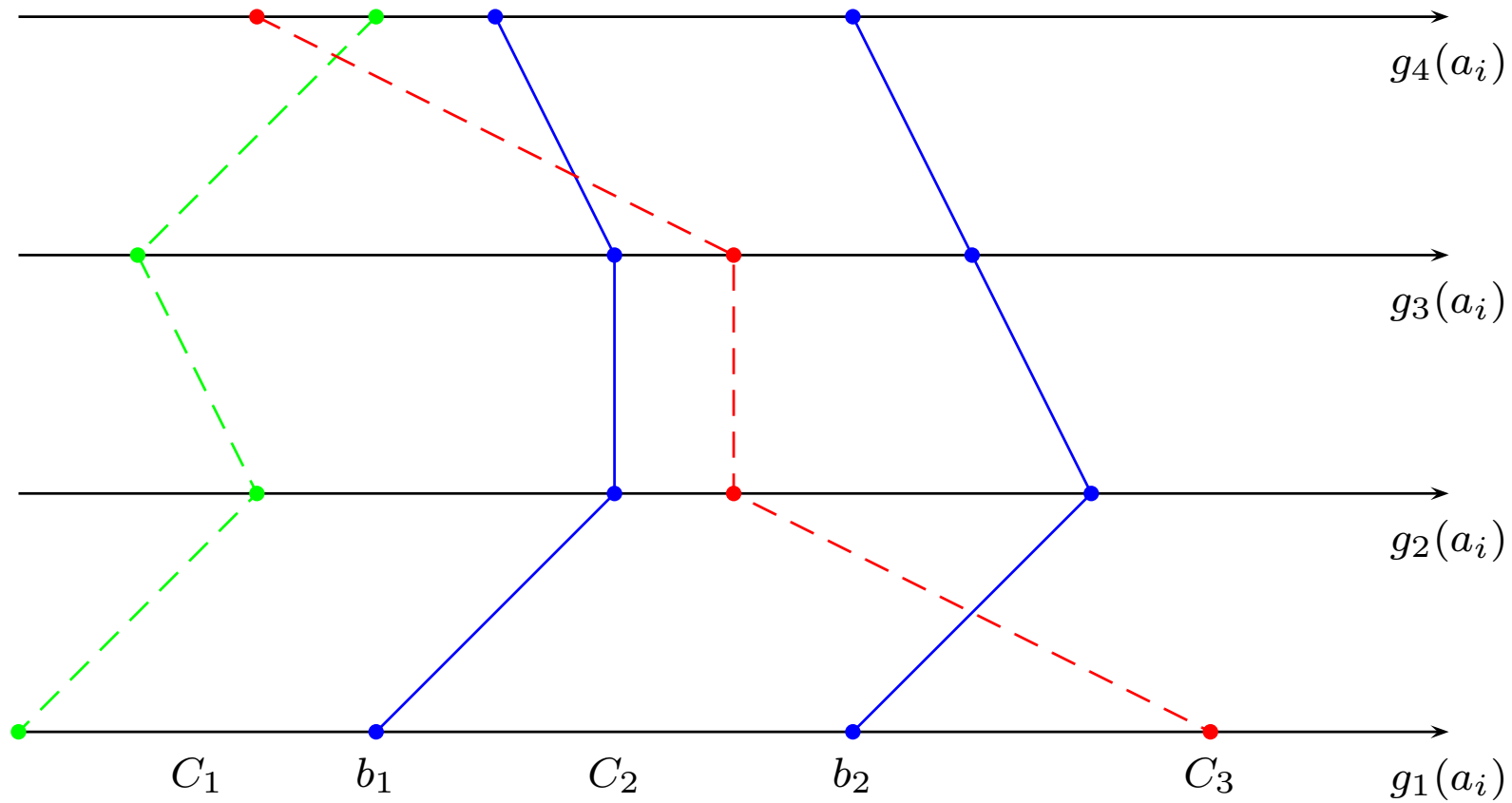


$$c(a_i, b_h) = \sum_{j \in \mathcal{J}} w_j c_j(a_i, b_h). \tag{6}$$

The OUTRANKING BASED METHODS : ELECTRE (DISCORD)



The OUTRANKING BASED METHODS : ELECTRE TRI



The OUTRANKING BASED METHODS : PROMETHEE

1. As an outranking method, PROMETHEE preference structures are based on **pairwise comparisons**.
2. When comparing two actions a and b on criterion g_j the **difference** of their **evaluations** is considered :

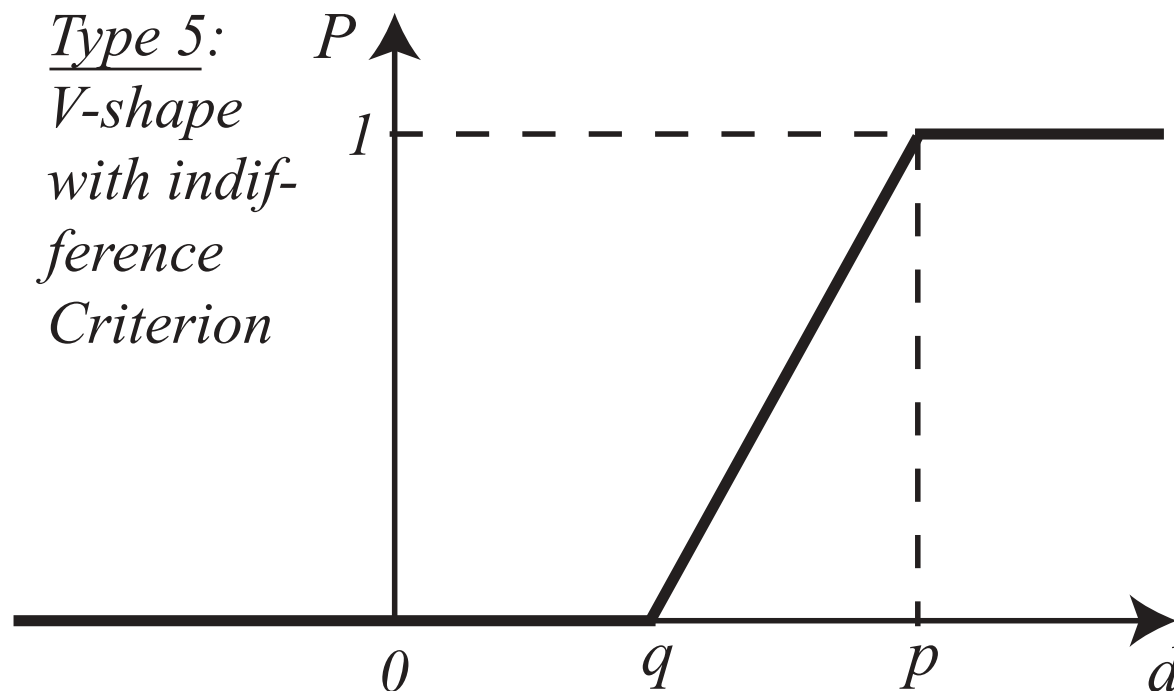
$$d_j(a, b) = g_j(a) - g_j(b)$$

3. This preference can be defined through a **preference function** in the following way,

$$P_j(a, b) = f_j(d_j(a, b)), \quad \forall a, b \in A$$

The OUTRANKING BASED METHODS : PROMETHEE

The pair $(g_j, P_j(a, b))$ is the *generalized criterion* associated with criterion g_j , for all $j \in F$.



The OUTRANKING BASED METHODS : PROMETHEE

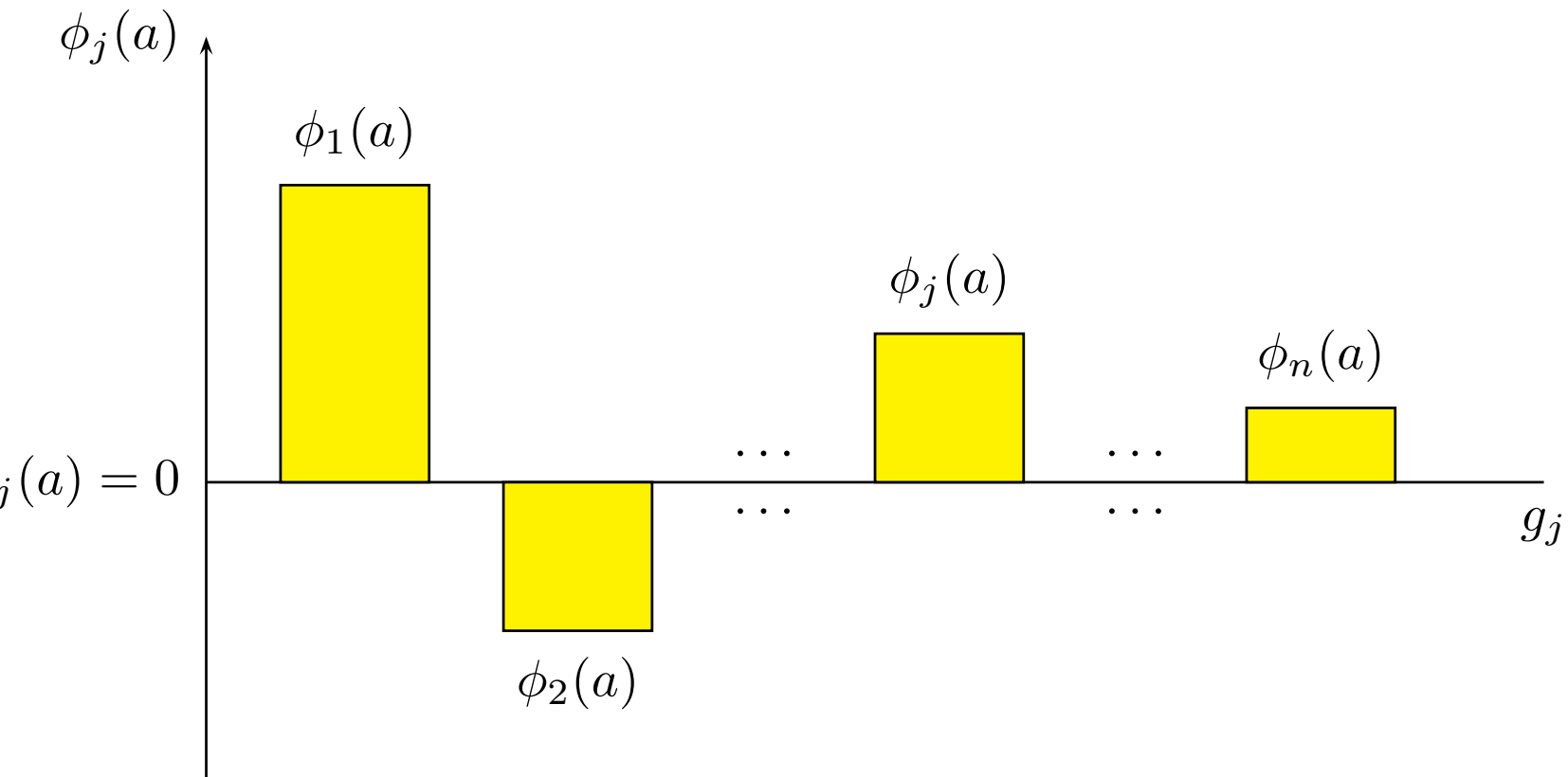
1. **Principle** : the quantification of how an action a *outranks* all the remaining $(m - 1)$ actions and how a is *outranked* by the other $(m - 1)$ actions.
2. This idea leads to the definition of the *positive*, *negative* and *net outranking flows*
3. For each action $a \in A$, we can also define the net flow for each criterion separately as follows,

$$\phi_j(a) = \frac{1}{m-1} \sum_{x \in A} \left(P_j(a, x) - P_j(x, a) \right)$$

4. Consequently, the multicriteria PROMETHEE net flow can also be defined in the following way,

$$\phi(a) = \sum_{j \in F} \phi_j(a) w_j$$

The OUTRANKING BASED METHODS : PROMETHEE



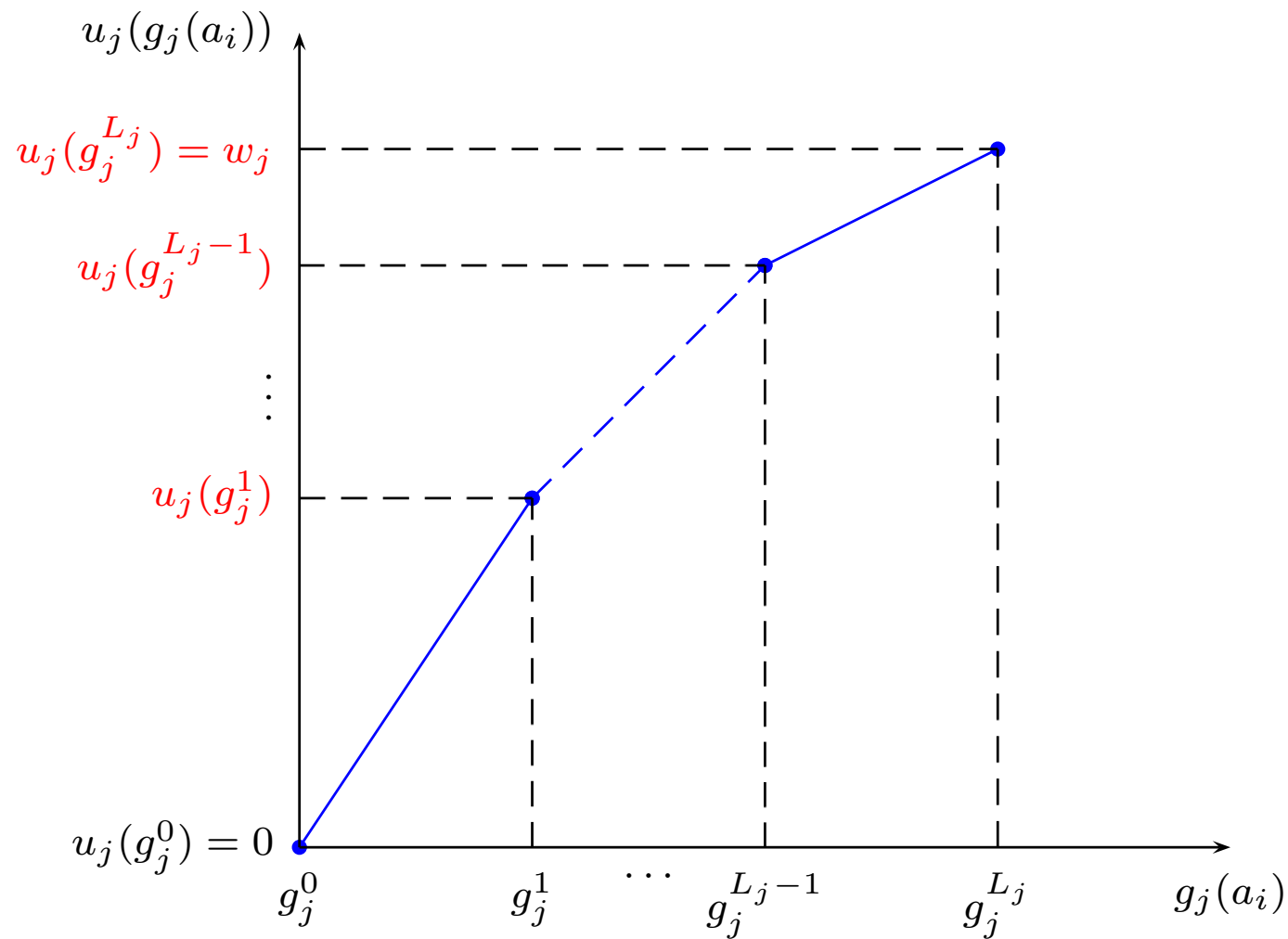
Utility and value theory based methods : MAUT

$$u(a_i) = \sum_{j \in F} w_j u_j(g_j(a_i)) \in [0, 1]$$

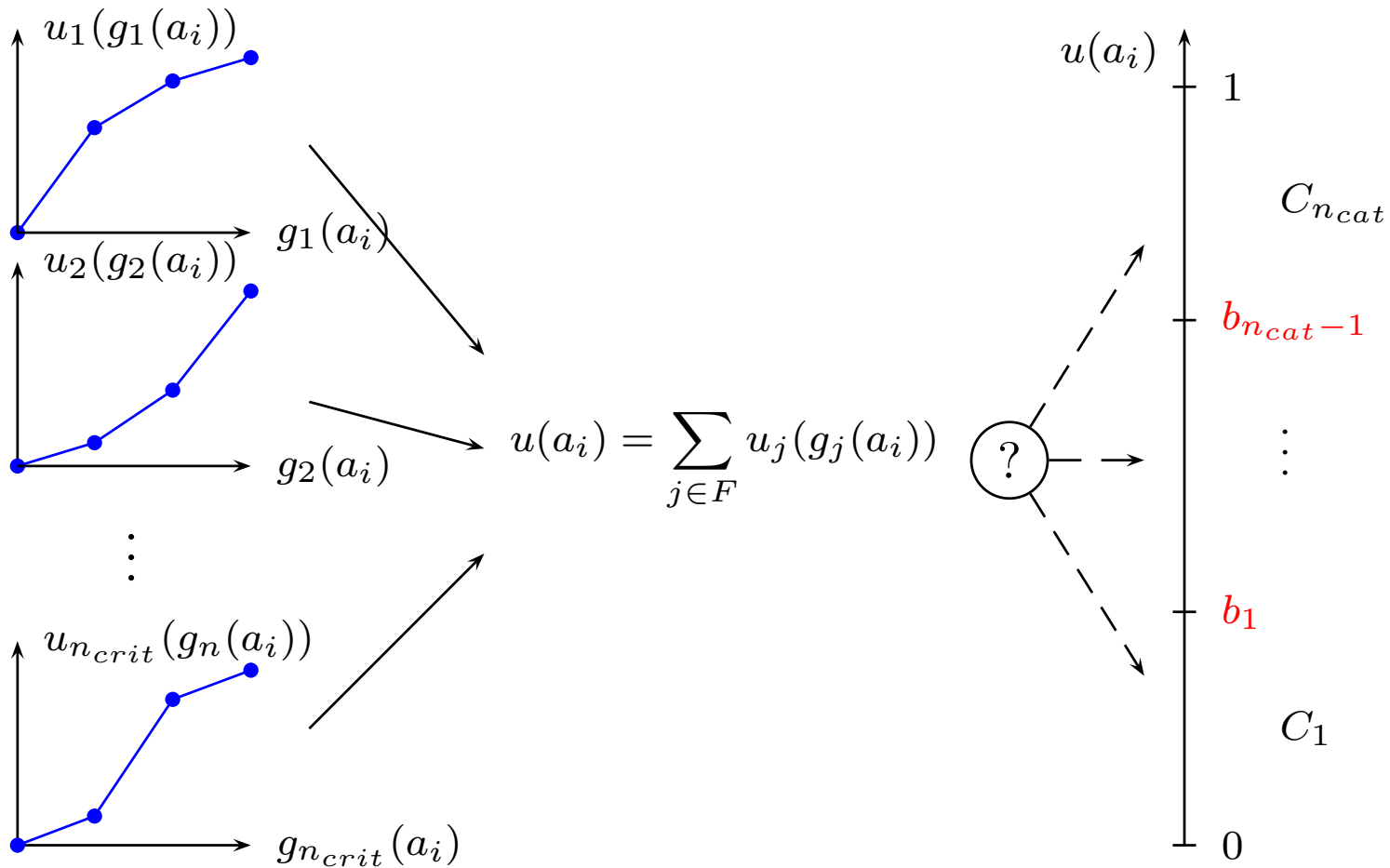
is the **comprehensive utility** of alternative a_i , where

1. $u_j(g_j(a_i)) \in [0, 1], \forall j \in F$
2. $\sum_{j \in F} w_j = 1$
3. $w_j \in]0, 1[, \forall j \in F$

Utility and value theory based methods : UTA



Utility and value theory based methods : UTA(DIS)



Utility and value theory based methods : AHP-ANP

Main points (according to T. Saaty) :

1. The Analytic Hierarchy Process (AHP) and its generalization to dependence and feedback, the Analytic Network Process (ANP), are theories of relative **measurement of intangible criteria**.
2. With this approach to relative measurement, **a scale of priorities** is derived from pairwise comparison measurements only after the elements to be measured are known.
3. In the AHP **paired comparisons** are made with **judgments using numerical values** taken from the AHP absolute fundamental **scale of 1-9**.

Utility and value theory based methods : AHP-ANP

4. A scale of relative values is derived from all these paired comparisons and it also belongs to an absolute scale that is invariant under the identity transformation like the system of real numbers.
5. The AHP/ANP is useful for making multicriteria decisions involving benefits, opportunities, costs and risks.

Utility and value theory based methods : MACBETH

- Firstly, through an **(ordinal) judgement** on their relative attractiveness ; and,
- Secondly, (if the two actions are not considered to be equally attractive), through a **qualitative judgement** about the difference of attractiveness between the most attractive of the two actions and a third one.

Seven **semantic categories** of difference of attractiveness are considered,

- 1) “null”,
- 2) “very weak”,
- 3) “weak”,
- 4) “moderate”,
- 5) “strong”,
- 6) “very strong”,
- 7) “extreme”.

Non-classical MCDA Approaches : Decision rule approach

Main features :

1. It is a methodology based on **preference modelling** in terms of “if ..., then ...” decision rules.
2. **Assumption** : The decision maker (DM) accepts to give preferential information in terms of **examples of decisions** and looks for simple rules justifying her decisions.
3. An important **advantage** : The possibility of **handling inconsistencies**.
4. The methodology is based on the **principle of dominance**.
5. The set of decision rules constituting the preference model is induced from the preferential information using a **knowledge discovery** technique.
6. The **mathematical basis** of the decision rule approach to MCDA is the **Dominance-based Rough Set Approach**.

Non-classical MCDA Approaches : Fuzzy measures and integrals

Main aspects :

1. Methods based on :
 - (a) Fuzzy measures (capacities)
 - (b) Fuzzy integrals, namely the Choquet and the Sugeno integrals
2. Interaction between criteria
3. Bi-polar scales
4. 2nd order measures.

Non-classical MCDA Approaches : SMAA

Six aspects should be taken into account when building a SMAA model :

1. **Problem Statement**. Ex : Ranking
2. **Preference model**. Ex : Weights
3. **Aggregation procedure** : Ex : Outranking procedure.
4. **Method**. Ex : SMAA-3
5. **Descriptive measures** (SMAA-3)
 - (a) Acceptability indices
 - (b) Central weights vectors
6. **Aggregate measures**. No aggregate measures for SMAA-3.

Software

1. **SRF**
2. **Choosing**
 - ELECTRE I, IV, IS
3. **Ranking**
 - MAUT
 - UTA, UTADIS, UTA-GMS, **GRIP**
 - MACBETH
 - ELECTRE II
 - ELECTRE III-IV, **New Versions**
 - PROMETHEE
4. **Sorting**
 - ELECTRE TRI, **TRICLAS, SMAA-TRI, IRIS**
 - **PROMETHEE TRI**
 - UTADIS

Multiobjective mathematical programming : Linear model

$$\max z_1 = c^1 x$$

$$\vdots$$

$$\max z_n = c^n x$$

$$s.t. \quad x \in X$$

- Identify the set of all non-dominated solutions
- Identify a particular set of non-dominated solutions
- Use an interactive method

Multiobjective mathematical programming : Linear model

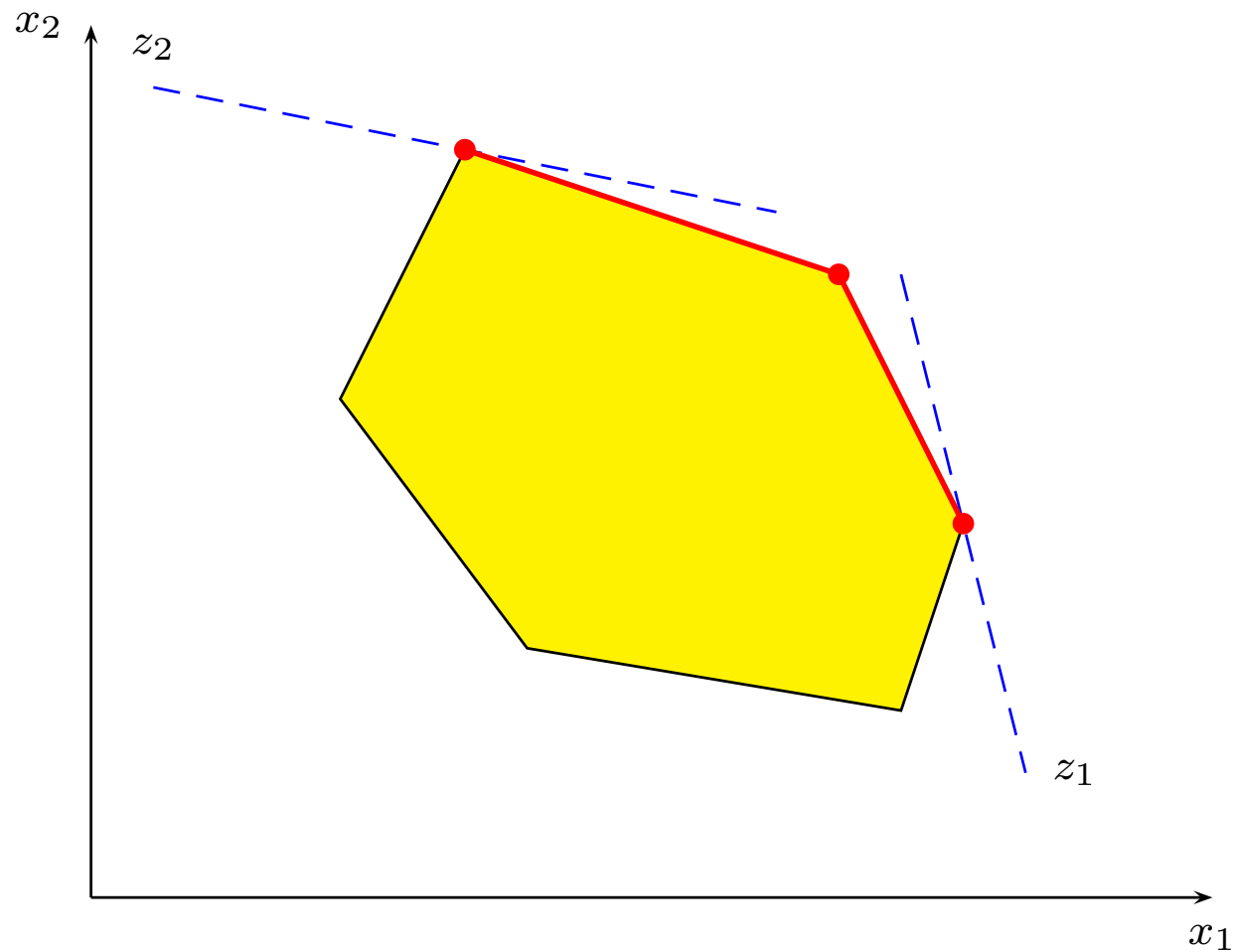


FIG. 1 – Multiple Objective Linear Programming

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