GRIP: A Generalized Regression Method with Intensities of Preferences for Ranking Alternatives Evaluated on Multiple Criteria

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Introduction

- We present a multiple criteria decision support methodology called GRIP (Generalized Regression with Intensities of Preference).
- The methods was proposed first for the problem of multiple criteria ranking of a finite set of actions.
- But, it can also be applied to interactive multi-objective optimization.
- And used for the multiple criteria sorting problem

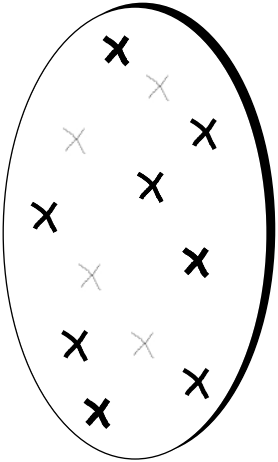
Introduction

- GRIP builds a set of additive value functions compatible with information about preferences of the Decision Maker (DM).
- The preference information is composed of a preference relation on a set of reference actions (partial preorder), and some intensities of preference, both with respect to single criteria or with respect to comprehensive (holistic) evaluations.
- It constructs not only the preference relation in the considered set of actions, but it also gives information about intensities of preference for pairs of actions from this set for a given Decision Maker (DM).
- Distinguishing necessary and possible consequences of preference information on the all set of actions, GRIP answers questions of robustness analysis.

Problem statements

- Choosing, from a set of potential alternatives, the best alternative or a small subset of the best alternatives (GRIP-MOO).
- Ranking the alternatives from the best to the worst (the ranking can be complete or not) (UTA, UTA^{GMS}, and GRIP)
- Assigning alternatives to pre-defined and ordered categories (UTADIS, UTADIS^{GMS}, and GRIPDIS).

Choice problem statement



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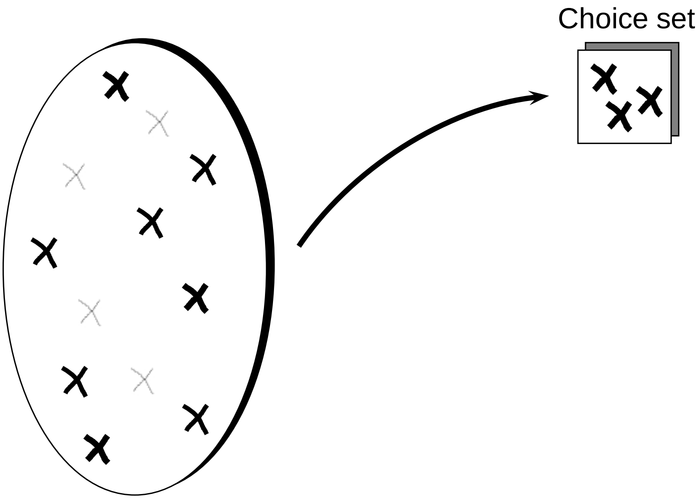

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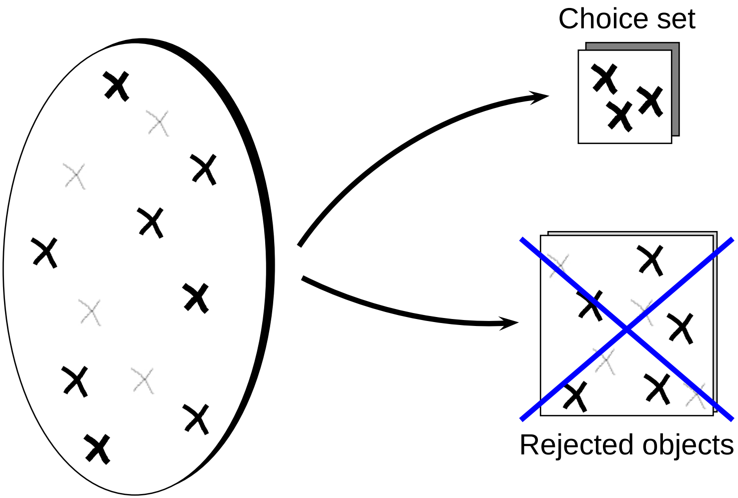
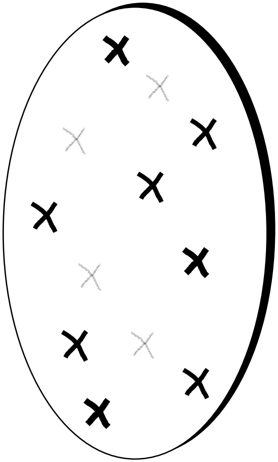


Ranking problem statement



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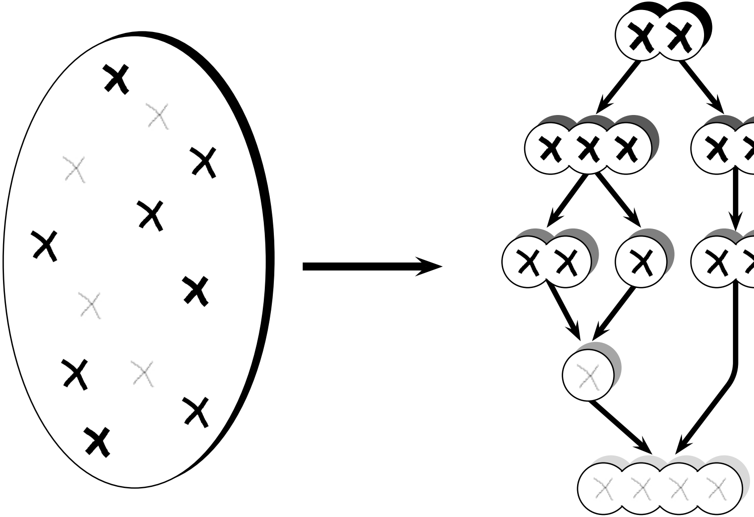
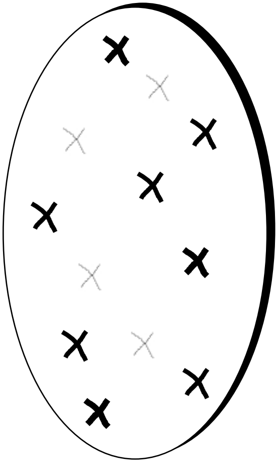


Sorting problem statement



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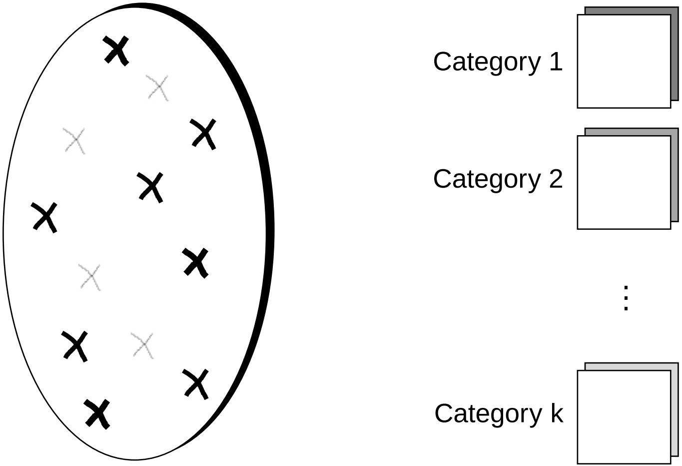

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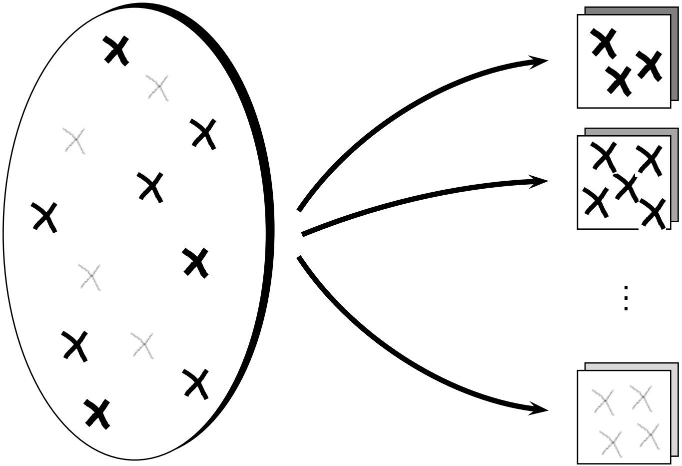


Software

- Choosing: GRIP-MOO, ELECTRE IS, ...
- Ranking: UTA, UTA^{GMS}, GRIP, MACBETH, AHP, ELECTRE III-IV, PROMETHEE, ...
- Classification: UTADIS, $UTADIS^{GMS}$, GRIPDIS, ELECTRE TRI, ...

Ordinal regression paradigm

- Traditional aggregation paradigm: The criteria aggregation model is first constructed and then applied on set A to get information about the comprehensive preference
- Disaggregation-aggregation (or ordinal regression) paradigm: Comprehensive preferences on a subset $A^R \subset A$ is known a priori, and a consistent criteria aggregation model is inferred from this information to be applied on set A.
- In our case, the preference model is a set of additive value functions compatible with a non-complete set of pairwise comparisons of some reference alternatives and information about comprehensive and partial intensities of preference

Elementary notation

- $A = \{a_1, a_2, \ldots, a_i, \ldots, a_m\}$ is finite set of alternatives,
- $g_1, g_2, \ldots, g_j, \ldots, g_n$ are the n criterion functions, F is the set of criteria indices,
- $g_j(a_i)$ is the evaluation of the alternative a_i on criterion g_j,
- G_j is the domain of criterion g_j, $G = \prod_{j \in F} G_j$ the evaluation space,
- $\succsim$ is the weak preference (outranking) relation on G: for each $x, y \in G$:
  - $x \succsim y \;\Leftrightarrow$ "x is at least as good as y",
  - $x \succ y \;\Leftrightarrow [x \succsim y$ and $not(y \succsim x)]$ "x is preferred to y",
  - $x \sim y \;\Leftrightarrow [x \succsim y$ and $y \succsim x]$ "x is indifferent to y".

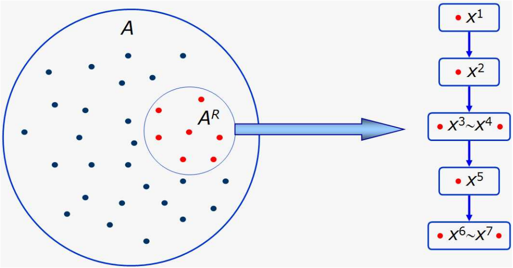
A remind on the UTA method (1)

- For each g_j, $G_j = [\alpha_j, \beta_j]$ is the criterion evaluation scale, $\alpha_j \leq \beta_j$,
- U is an additive value function on G: for each $x \in G$, $U(x) = \sum_{j \in F} u_j[g_j(x)]$,
- u_j are non-decreasing marginal value functions, $u_j : G_j \mapsto \mathbb{R}$, $\forall j \in F$

Reminder on the UTA method (2)

- The preference information is given in the form of a complete pre-order on a subset of reference alternatives $A^R \subseteq A$, called reference pre-order.

- $A^R = \{a_1, a_2, ..., a_{m1}\}$ is rearranged such that $a_k \succsim a_{k+1},\ k = 1, ..., m_1 - 1$, where $m_1 = |A^R|$.



Reminder on the UTA method (3)

- The inferred value of each $a \in A^R$ is :

$$U(a) + \sigma^+(a) - \sigma^-(a),$$

- In UTA , the marginal value functions u_i are assumed to be piecewise linear, so that the intervals $[\alpha_i, \beta_i]$ are divided into $\gamma_i \geq 1$ equal sub-intervals

$$[x_i^0, x_i^1], [x_i^1, x_i^2], \ldots, [x_i^{\gamma_i - 1}, x_i^{\gamma_i}],$$

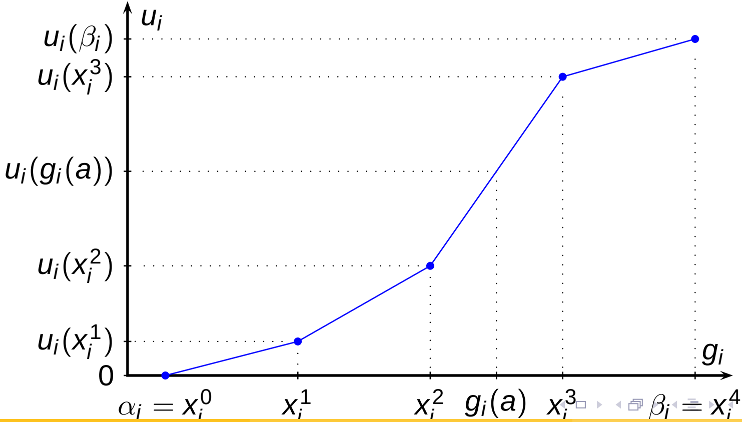
where,

$$x_i^j = \alpha_i + \frac{j(\beta_i - \alpha_i)}{\gamma_i}, j = 0, \ldots, \gamma_i, i = 1, \ldots, n.$$

Reminder on the UTA method (4)

- The piecewise linear additive model is completely defined by the marginal values at the break points, i.e.

$$u_i(x_i^0) = u_i(\alpha_i),\ u_i(x_i^1), u_i(x_i^2), \ldots, u_i(x_i^{\gamma_i}) = u_i(\beta_i).$$



The UTA^{GMS} method: Main features (1)

UTA^{GMS} method generalizes the UTA method in two aspects:

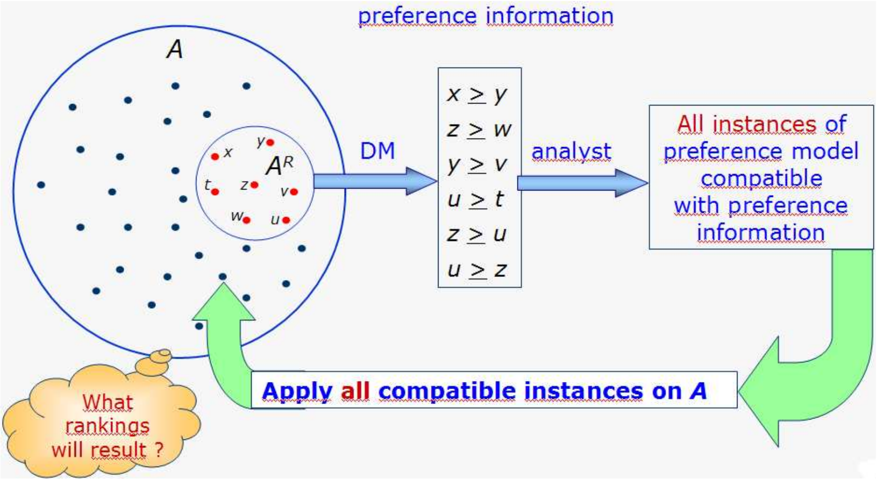
- It takes into account all additive value functions compatible with indirect preference information, while UTA is using only one such function.
- The marginal value functions are general monotone non-decreasing functions, and not piecewise linear, as in UTA.

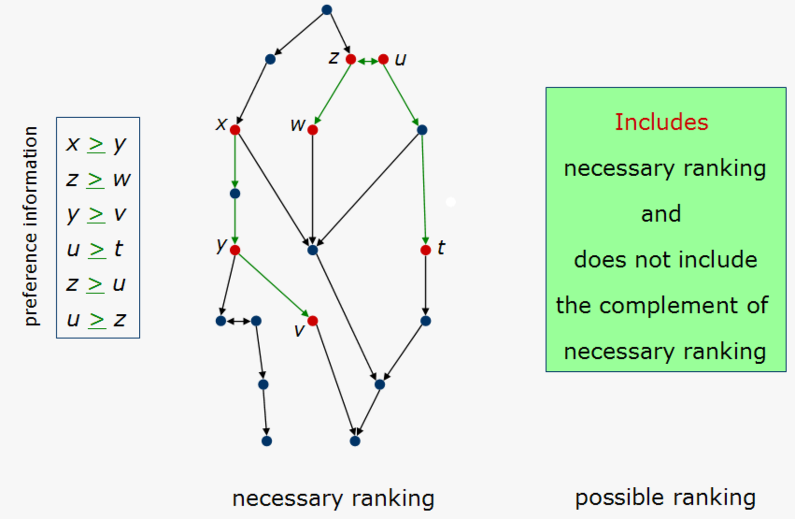
The UTA^{GMS} method: Main features (2)

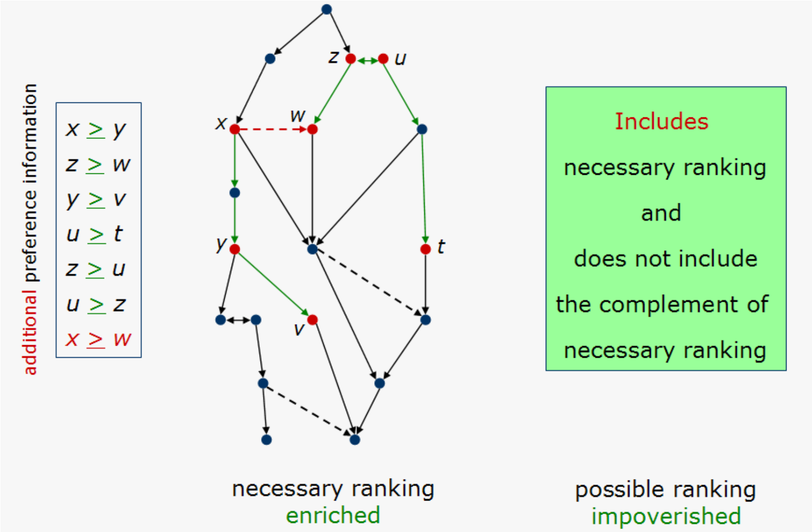
The method produces two rankings in the set of alternatives A, such that for any pair of alternatives $a, b \in A$,

- In the *necessary order*, a is ranked at least as good as b if and only if, $U(a) \geq U(b)$ for all value functions compatible with the preference information.
- In the *possible order*, a is ranked at least as good as b if and only if, $U(a) \geq U(b)$ for at least one value function compatible with the preference information.

The UTA^{GMS} method: Main features (3)




preference information
x ≥ y
z ≥ w
y ≥ v
u ≥ t
z ≥ u
u ≥ z
z
u
x
w
y
t
v
necessary ranking
Includes
necessary ranking
and
does not include
the complement of
necessary ranking
possible ranking


additional preference information
x ≥ y
z ≥ w
y ≥ v
u ≥ t
z ≥ u
u ≥ z
x ≥ w
z
u
x
w
y
t
v
Includes
necessary ranking
and
does not include
the complement of
necessary ranking
necessary ranking
enriched
possible ranking
impoverished

The GRIP method: Main features

GRIP extends both UTA and UTA^{GMS} methods by adopting all features of UTA^{GMS} and taking into account additional preference information in form of comparisons of intensities of preference between some pairs of reference alternatives. For alternatives $x, y, w, z \in A$, these

comparisons are expressed in two possible ways (not exclusive),

1) Comprehensively, on all criteria, "x is preferred to y at least as much as w is preferred to z".

2) Partially, on each criterion, "x is preferred to y at least as much as w is preferred to z, on criterion $g_i \in F$".

The GRIP method: Preference Information

DM is expected to provide the following preference information,

- A partial pre-order $\succsim$ on A^R whose meaning is: for $x, y \in A^R$

$$x \succsim y \Leftrightarrow x \text{ is at least as good as } y.$$

- A partial pre-order $\succsim^*$ on $A^R \times A^R$, whose meaning is: for $x, y, w, z \in A^R$,

$$(x, y) \succsim^* (w, z) \Leftrightarrow \; x \text{ is preferred to } y \text{ at least as much as } w \; .$$

  is preferred to z

- A partial pre-order $\succsim_i^*$ on $A^R \times A^R$, whose meaning is: for $x, y, w, z \in A^R$,

$$(x, y) \succsim_i^* (w, z) \Leftrightarrow \; x \text{ is preferred to } y \text{ at least as much as } w$$

  is preferred to z on criterion g_i, $i \in I$.

- It is easy to incorporate other kind of preference information like local trade-offs.

Note: Intensity of preferences can also be handled by a MACBETH like procedure.

The GRIP method: Results

- a necessary ranking $\succsim^N$, for all pairs of actions $(x, y) \in A \times A$;
- a possible ranking $\succsim^P$, for all pairs of actions $(x, y) \in A \times A$;
- a necessary ranking $\succsim^{*N}$, with respect to the comprehensive intensities of preferences for all $((x, y), (w, z)) \in A \times A \times A \times A$;
- a possible ranking $\succsim^{*P}$, with respect to the comprehensive intensities of preferences for all $((x, y), (w, z)) \in A \times A \times A \times A$;
- a necessary ranking $\succsim_i^{*N}$, with respect to the partial intensities of preferences for all $((x, y), (w, z)) \in A \times A \times A \times A$ and for all criteria g_i, $i \in I$;
- a possible ranking $\succsim_i^{*P}$, with respect to the partial intensities of preferences for all $((x, y), (w, z)) \in A \times A \times A \times A$ and for all criteria g_i, $i \in I$.

The GRIP method: Summary of main features

- It is using a general additive value function to represent preferences: a feasible space of value functions is identified and any additive function belonging to that set is called a compatible value function.

- The preference information can be given as a partial preorder on the set of reference actions.

- It can deal with intensities of preferences, comprehensively or partially.

- Two preference relations - necessary and possible - are considered to take into account certain or conceivable preferences, respectively.

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The GRIP method: Summary of main features (cont.)

- It can represent incomparability between actions: the necessary preference is not complete, in general.
- It provides robust conclusions: the necessary and possible preference relations are based on all compatible value functions, rather than on only one or few among the many possible functions, as it is usual in MCDA.
- It permits to detect inconsistent preference information: once the ordinal regression fails to find any compatible value function, the inconsistent pairwise comparisons can be detected to remove this impossibility.
- It can be used in an interactive procedure: the DM can modify the preference information verifying its impact on the preference relations in the set of considered actions.

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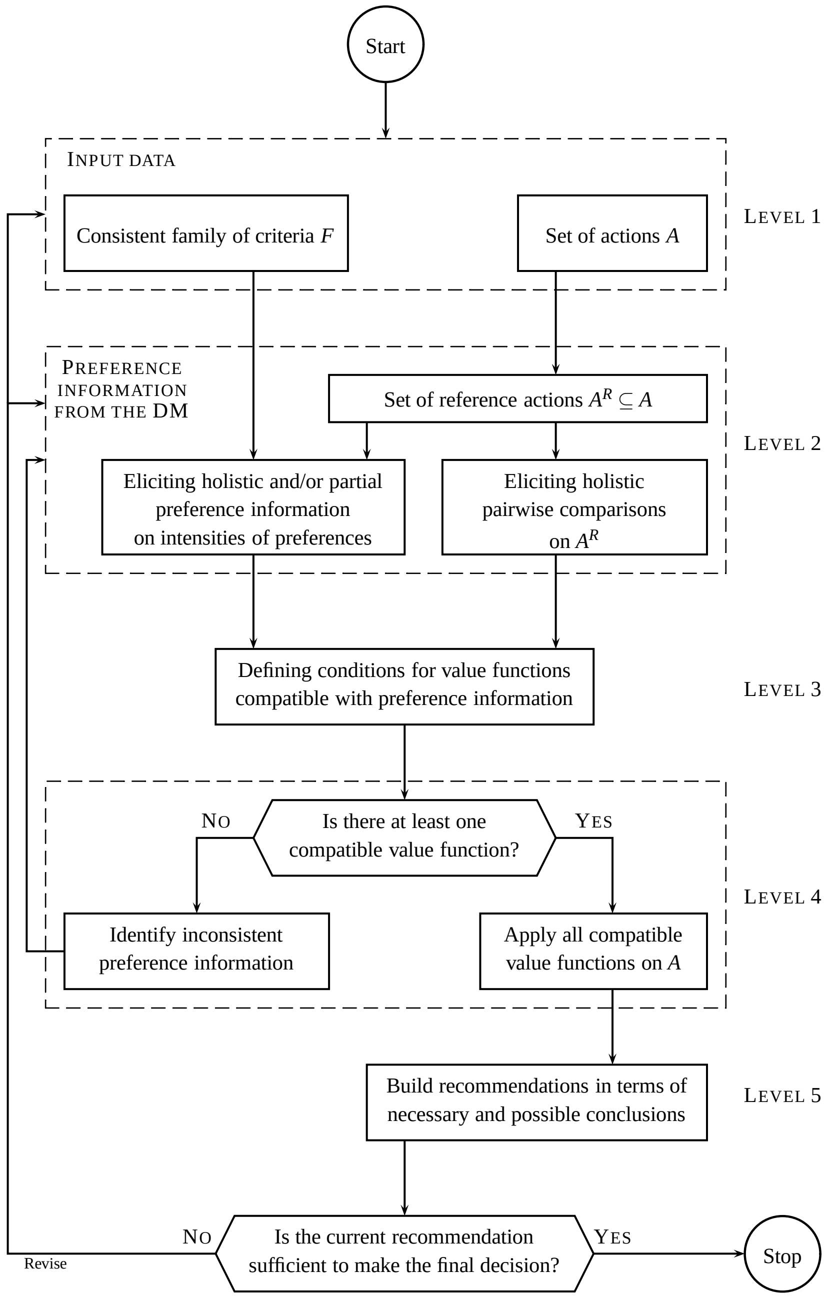

Figure 1: GRIP decision support process

The GRIP *versus* MACBETH

- Both deal with qualitative judgements.
- both need a set of comparisons of actions or pairs of actions to work out a numerical representation of preferences, however, MACBETH depends on the specification of two characteristic levels on the original scale, “neutral” and “good”, to obtain the numerical representation of preferences, while GRIP does not need this information.
- GRIP adopts the “disaggregation-aggregation” approach and, therefore, it considers mainly holistic judgements relative to comparisons involving jointly all the criteria, which is not the case of MACBETH.
- GRIP is more general than MACBETH since it can take into account the same kind of qualitative judgments as MACBETH (the difference of attractiveness between pairs of actions) and, moreover, the intensity of preferences of the type “x is preferred to y at least as much as z is preferred to w”.

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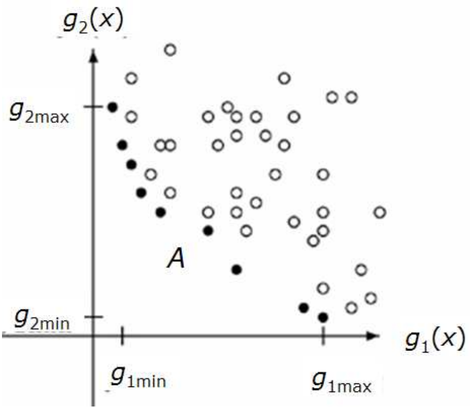
Introduction

- Multiple criteria mathematical programming is mainly focussing on generating of the exact or approximate Pareto frontier
- The problem solved is handled rather as a mathematical problem than as a decision aiding one.
- Such an observation led us to investigate why no more particular attention is devoted to interactive methods.
- A relevant related question is: How to incorporate preference information within a mathematical programming problem context?

Incorporate preference in MOCO problems



A preference model discriminates among alternatives in the Pareto front

Introduction (cont)

- What did we observe? A missing link between multiple criteria mathematical programming and decision aiding that, in our view, should be strengthen.
- How can it be mitigated? By changing the paradigm in the way preference information is obtained; the ordinal regression paradigm seems suitable for such purposes.
- Why is it so important? A huge amount of real-world situations require the use of interactive methods to get solutions to be implemented.

Main features of the proposed approach

$$\begin{array}{ll} \max z_1(x) & \\ \max z_2(x) & \\ \vdots & \\ \max z_n(x) & \\ s.t. & x \in X \end{array}$$

- Identify the (a) set of non-dominated solutions
- Elicit a preference information from the DM
- Construct a preference model on the set of non-dominated solutions
- Elaborate recommendations

Main features of the proposed approach

1. Compute a set A of Pareto alternatives (either the entire exact set, a sub set, or even an approximation of the Pareto frontier) of a MOCO problem

2. Elaborate a preference model on A, that should be grounded on a set of additive monotonically non-decreasing value functions (as in UTA^{GMS} and GRIP)

3. Interact progressively with the DM to elicit the preference model on A (that will provide a ranking on A, or support a choice in A)

Main features of the proposed approach (cont.)

4. The DM can provide information in various forms,
   - pairwise comparisons of $(a, b) \in A \times A$,
   - comparisons of intensities of preferences among pairs $(a, b), (c, d) \in A \times A$,
   - comparisons of the intensities of difference of preferences on a single criterion,
   - ...
5. The approach allows to elicit and refine progressively the preference model and at the same time enrich the partial pre-order on A
6. The approach can deal with situation in which the DM provide information that is not compatible with the additive value function (inconsistency resolution)

Features of the proposed approach

- Interaction with the DM based on simple and intuitive statements,
- The elicited preference information as well as the output of the method are of an ordinal nature,
- The graphical representation of the output rankings is well accepted by the DMs,
- The DM can observe the impact of the modification of the input information on the rankings,

Conclusions and future research

- We presented new developments of UTA-like methods.
- The methods can be applied to choosing (GRIP-MOO), ranking (GRIP), and sorting problem statements (GRIPDIS).
- GRIP is competitive to AHP and MACBETH method for ranking problems.
- A software on GRIP methodology is currently being implemented.
- There is a wide range of potential applications for the methodologies proposed in this talk.

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